EVALUATION OF RELIABILITY OF RUPTURE AND ROTATIONAL STRENGTH OF THE CERAMIC TURBINE WHEEL

HAIDER HADI JASIM

Chemical Engineering Department, College of Engineering,
Basrah University, Basrah, Iraq
e-mail: raidhani73@yahoo.com

Abstract

In this paper, Weibull uni-axial and multi-axial distribution function is applied to evaluate the reliability of the fracture strength of rotating turbine rotor wheel manufactured from ceramic material and have inner surface crack. Three cases are considered, first taking only the effect of rotational stresses, second taking the effect of rotational and thermal stresses in ceramic disc, and third taking the effect of rotational and thermal loading in ceramic blade. It was found that there is a convergence between results gotten from uni-axial and multi-axial distribution function, but multi-axial distribution function give small large in values result compared to uni-axial distribution function. The expected values of rupture strength of ceramic blade is higher than of that of disc material, therefore the failure occurs in blade first than in disc material in service survival.

حساب احتمالية الفشل واحتمالية مقاومة الانكسار لقرص تربيني دوار مصنوع من مواد سيراميك

حيدر هادي جاسم

في هذا البحث، تم تطبيق دوال ويبل أحادية التوزيع للبعد ومتعددة التوزيع للإبعاد لحساب احتمالية الفشل واحتمالية مقاومة الانكسار لقرص تربيني دوار مصنوع من مواد سيراميكية ومزود بريش يحتوي على شق داخلي. تم اخذ ثلاث حالات دراسية، في الحالة الأولى كان القرص معرض إلى اجهادات دورا نية فقط، أما الحالة الثانية فكان القرص معرض إلى اجهادات دورا نية وحرارية، أما الحالة الثالثة كان ريشة تربين معرضة إلى أحمال دورا نية وحرارية فقط. لقد أظهرت النتائج تقارب في القيم الناتجة باستخدام دوال ويبل أحادية التوزيع للبعد ومتعددة التوزيع للأبعاد، لكن الدوال متعددة التوزيع تعطي قيم لاحتمالية الفشل أعلى بقليل من دوال ويبل أحادية التوزيع. القيمة المتوقعة لانكسار نتيجة السرعة الدورانية للريشة أعلى بكثير مما في القرص، لذا فان الفشل يحدث في الريشة ثم في القرص في التطبيق العملي.

Nomenclature

R_i : Inner radius of disc.

R_o : Outer radius of disc.

h: Thickness of disc.

K: Parameter Constant.

dA : Area element of the unit sphere.

dV : Volume element of unite sphere.

$S(B_o)$: Probability of survive.

B_o: Volume or surface area of a body.

m: Weibell modulus.

P_o and P: Source and integral points, respectively.

u_i and t_i : Displacement and traction on a boundary node P respectively.

a_k : Rotational loading.

ΔT : Temperature change

E: Modulus of elasticity.

ν : Poisson's ratio.

ω : Angular velocity (rad / s).

ρ : Mass density (kg / m^3).

σ : Applied stress.

σ_u : Stress below which there is a zero probability of failure.

σ_o : Mean strength of material.

σ_t : Circumferential stress.

σ_r : Radial stress.

$E(\sigma_{max.})$: The expected value of fracture rotating speed.

μ : Shear modulus.

α : Coefficient of thermal expansion.

φ and ϕ : angle between crack and coordinates.

β : Beta functions.

1-Introduction

Ceramic turbine wheel and ceramic blades are one of the key parts to increasing the efficiency of turbocharger and gas turbine engine due to resist high temperature and high strength [1].

Recently, the use of high-speed ceramic wheel increase rapid, and it is necessary to study the rupture strength using probability theory. Most studies focused on using Weibull uniaxial distributions function. In general, in the domain of ceramic wheel where tensile stresses are dominant, in the weakest region the fracture can occurs. Robert H. [2] used Weibull uniaxial distribution function to life prediction at high temperature of silicone carbide ceramic wheel. Licht H. R. [3] used experimental approaches to determine the failure probability of Si_3N_4 ceramic wheel. Nobuo Kamiya [4] developed a technique involving taking moment photographs from two and three directions at failure of ceramic radial rotor to determine the poison of crack. Takatori K. [5] found the optimum condition to fabricate silicon nitride radial turbine wheel for gas turbine engine without flaw by injection molding. Duffy S. F. [6] studied the strength and reliability of ceramic disc used at room and high temperature using experimental method. Yotaro M. [7] developed a new distribution function depending on Weibull model to expected values of the rupture strength of brittle rotating disc.

In this paper, Weibull multi-axial distribution model is used in analysis of ceramic turbine wheel having blades to evaluate the reliability of rupture. Three cases are taken in analysis, ceramic disc under rotation loading only, ceramic disc under rotation and thermal loading, and ceramic blade under rotation and thermal loading.

2-Weibull Analysis

The key element in the design and fabrication of a component pertains it is reliability (usually determined by a safety or economic consideration), it is important to know the statistical probability of a given fracture event. According to Weibull, the two parameter distribution function $S(B_o)$ when a body subjected to a uni-axial tensile stress (σ) is given by [7] :

$$S(B_o) = e^{-B_o\left(\frac{\sigma-\sigma_u}{\sigma_o}\right)^m} \quad \ldots (1)$$

For brittle ceramic material $\sigma_u = 0$

The risks of rupture (R) define by the following equation:

$$R = 1 - e^{-B_o\left(\frac{\sigma}{\sigma_o}\right)^m} \quad \ldots (2)$$

Weibull also has a heuristically obtained the following multi-axial distribution function by taking the direction of the cracks at every point in the body into account for multi-axial stress state ($\sigma_1, \sigma_2, \sigma_3$) as shown in Fig.1 [8]:

$$R = 1 - e^{-\int_V \left(K \int_A \sigma_n^m dA\right) dV} \quad \ldots (3)$$

Where,

σ_n : Normal stress on the crack plane and is define by:

$$\sigma_n = \cos^2\phi(\sigma_1 \cos^2\varphi + \sigma_2 \sin^2\varphi) + \sigma_3 \sin^2\phi \quad \ldots (4)$$

K: Parameter Constant and define by the following equations:

$$K = \frac{(2m+1)}{2\pi\sigma_o^m} \quad \ldots (5)$$

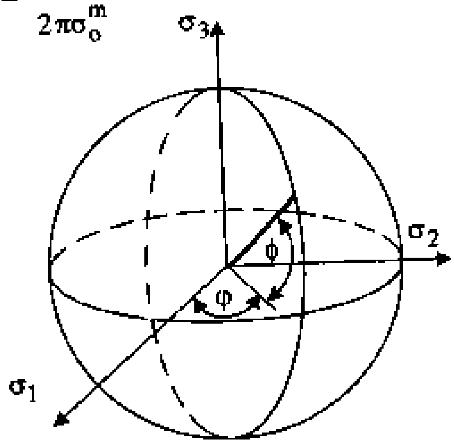

Fig.1 Unit sphere

3- The Expected Rupture Strength of a Hollow Ceramic Disc Under Rotation Loading.

For a hollow ceramic disc rotating at angular velocity (ω), the circumferential stress (σ_t) and radial stress (σ_r) are defined by [9]:

$$\sigma_t = \frac{\rho\omega^2}{8}[(3+\nu)(R_i^2 + R_o^2 + \frac{R_i^2R_o^2}{r^2}) - (1+3\nu)r^2] \quad \dots (6)$$

$$\sigma_r = \frac{\rho\omega^2}{8}(3+\nu)(R_i^2 + R_o^2 + \frac{R_i^2R_o^2}{r^2} - r^2) \quad \dots (7)$$

Since $\sigma_t > \sigma_r$ at any radius in the disc, and maximum circumference stress occurs at inner radius then, the maximum stress ($\sigma_{max.}$) define by:

$$\sigma_{max.} = (\sigma_t)_{r=R_i}$$

$$\sigma_{max.} = \frac{\rho\omega^2}{4}[(3+\nu)R_o^2 + (1-\nu)R_i^2] \quad \dots (8)$$

In the following sections, propose risk of rupture by using distribution functions described in Eq.1 and Eq.2.

A- In the case of taking only internal cracks into consideration (uni-axial distribution function), for a hollow disc, defining the function $f_1(r)$ is [6]:

$$f_1(r) = \frac{\sigma_t}{\sigma_{max.}}$$

$$f_1(r) = \frac{(R_o^2 + R_i^2 + \frac{R_o^2R_i^2}{r^2}) - \frac{(1+3\nu)r^2}{(3+\nu)}}{2(R_o^2 + \frac{(1-\nu)}{(3+\nu)}R_i^2)} \quad \dots (9)$$

And,

$$R(\sigma_{max.}) = 1 - e^{-B_o(\frac{\sigma_{max.}}{\sigma_o})^m} \quad \dots (10)$$

Where,

$$B_o = 2\pi h \int_{R_i}^{R_o} f(r)^m r dr \quad \dots (11)$$

B- In the case of taking only internal cracks into consideration (multi-axial distribution function), defining $f_2(r)$ in a hollow disc as:

$$f_1(r) = \frac{\sigma_r}{\sigma_{max}}$$

$$f_1(r) = \frac{(R_o^2 + R_i^2 - \frac{R_o^2R_i^2}{r^2}) - \frac{r^2}{(3+\nu)}}{2(R_o^2 + \frac{(1-\nu)}{(3+\nu)}R_i^2)} \quad \dots (12)$$

And,

$$R(\sigma_{max.}) = 1 - e^{-B_1 K(\frac{\sigma_{max}}{\sigma_o})^m} \quad \dots (13)$$

Where,

$$B_1 = \Phi \int_{R_i}^{R_o} \int_0^{\frac{\pi}{2}} [f_1(r)\cos^2\varphi + f_2(r)\sin^2\varphi]^m r d\varphi dr \quad \dots (14)$$

Where,

$$\Phi = \frac{4\pi h}{\beta(m+\frac{1}{2},\frac{1}{2})}$$

The β (Beta function) define by equation:

$$\beta(\chi,\lambda) = \int_0^1 x^{\chi-1}(1-x)^{\lambda-1}dx$$

χ and λ : Constants and $\chi > 0$, $\lambda > 0$.

4- The Expected Rupture Strength of a Hollow Ceramic Disc Under Rotation and Thermal Loading.

For a hollow disc subjected at inner surface to temperature T_i and outer surface to temperature T_o, and for steady state heat flow, the temperature distribution through disc is derived as followers [9]:

$$r\frac{dT}{dr} = c$$

$$\frac{dT}{dr} = \frac{c}{r}$$

$$T = b\ln(r) + a \qquad \dots (15)$$

Where a and b: Constants

The radial and tangential stresses are define by the following equations [9]:

$$\sigma_r = A - \frac{B}{r^2} - \frac{E\alpha T}{2(1-\nu)} - (3+\nu)\frac{\rho\omega^2 r^2}{8} \qquad \dots (16)$$

$$\sigma_t = A + \frac{B}{r^2} - \frac{E\alpha T}{2(1-\nu)} - \frac{E\alpha b}{2(1-\nu)} - (1+3\nu)\frac{\rho\omega^2 r^2}{8} \qquad \dots (17)$$

Where, A and B are constants and determine from the condition $\sigma_r = 0$ at $r = R_i$ and $r = R_o$.

The maximum stress occurs at inner radius and given by equations:

$\sigma_{max.} = \sigma_t$ at $r = R_i$

$$\sigma_{max} = \frac{E\alpha}{(1-\nu)(R_i^2 - R_o^2)}(T_i - T_o) + \frac{(3+\nu)}{4}\rho\omega^2 R_i^2(R_o^2 + 2) + \frac{E\alpha T_i}{(1-\nu)} + \frac{E\alpha b}{(1-\nu)} - \frac{(1+3\nu)}{8}\rho\omega^2 R_i^2 \qquad \dots (18)$$

The distribution function for uni-axial and multi-axial distribution is define by:

$$f_1(r) = \frac{\sigma_t}{\sigma_{max}} \quad \text{and} \quad f_2(r) = \frac{\sigma_r}{\sigma_{max}}$$

Then Eq.11 and Eq.12 are used for uni-axial and Eq.13 and Eq.14 for multi-axial distribution to determine the risk of rupture.

5- The Expected Value of Fracture Rotating Speed

The expected values of ($\sigma_{max.}$) for fracture of disc occurs can be calculate from the following equation [10]:

$$E(\sigma_{max.}) = \int_0^\infty (1 - R(\sigma_{max.}))d\sigma_{max.} \qquad \dots (19)$$

The expected values of (ω^2) which fracture occurs in uni-axial and multi-axial distribution function are giving by:

$$E(\omega^2) = \frac{4}{\rho((3+\nu)R_o^2 + (1-\nu)R_i^2)} * E(\sigma_{max.}) \quad \dots (20)$$

Fig.2 show a disc shape specimen and dimensions. Table1. show the mechanical properties and Weibull modules used in analysis [11].

Eq.12, Eq.14 and Eq.20 are solved using numerical gauss integration for high precision in results.

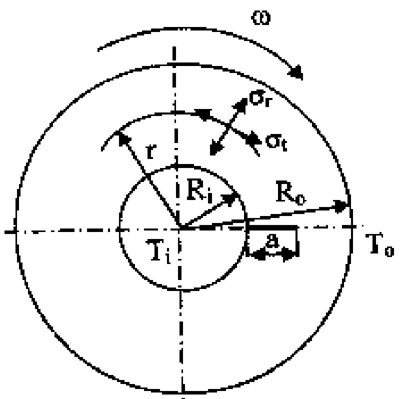

Fig.2 Disc specimen shape and dimensions.

The wheel has the following dimensions and conditions:

R_i = 40 mm, R_o = 142 mm, h = 40 mm

$T_i = 200^\circ C$, $T_o = 100^\circ C$,

$\omega = 5835 rad/s$,

a = 10 mm crack length

Table1. Mechanical properties of the Silicon Nitride ceramic disc with Y_2O_3 and $MgAl_2O_4$ addition.

Quantity	constants
E	294 GPa.
ν	0.28
ρ	3210 kg/m^3
α	$1.5 * 10^{-6} / ^\circ C$
m	4.2

6- Rotational Ceramic Blade

In the case of ceramic blade subjected to rotational loading, no analytical solution is found. Therefore, the numerical boundary element method (BEM) is used to determine the maximum stress in blade for better accuracy [12, 13]. Then can be used Eq.2 and Eq.3 to determine the risk of rupture in blade.

The boundary integral equation of an isotropic elastic body under thermal and centrifugal loads can be expressed as [14]:

$$u_j(P_o) = \int_L [U_{ij}(P,P_o)t_j(P) - T_{ij}(P,P_o)u_j(P)]dL(P) + \int_L [S_t(P,P_o)\Delta T - V_i(P,P_o)\frac{\partial T}{\partial n}]dL(P) + \int_L a_i(P,P_o)dL(P) \quad \ldots (21)$$

And,

$$T_{ij}(P,P_o) = \frac{-1}{4\pi(1-\nu)r}\{ \frac{\partial r}{\partial n}[(1-2\nu)\delta_{ij} + 2r_{,i}r_{,j}] + (1-2\nu)(n_i r_{,j} - n_j r_{,i}) \}$$

$$U_{ij}(P,P_o) = \frac{1}{8\pi\mu(1-\nu)} * [-(3-4\nu)\ln\frac{1}{r}\delta_{ik} + r_{,i}r_{,j}]$$

$$r_{,i} = \frac{\partial r}{\partial x}, \quad r_{,k} = \frac{\partial r}{\partial y}$$

$$S_i = \frac{-\alpha(1+\nu)}{4\pi(1-\nu)}[(\ln r + \frac{1}{2})n_i + \frac{\partial r}{\partial n}r_{,i}$$

$$V_i = \frac{-\alpha(1+\nu)}{4\pi(1-\nu)}[(\ln r + \frac{1}{2})r_{,i}\, r$$

$$a_i = \frac{-\rho\omega^2 r}{8\pi\mu}\{(2\ln r + \frac{1}{2})[\frac{\partial r}{\partial n}y_i - \frac{1}{2(1-\nu)}y_j r_{,j} n_i] - \frac{1-2\nu}{2(1-\nu)}rn_i \ln r\}$$

$q = \frac{\partial T}{\partial n}$ the temperature derivative on the outer normal direction of the boundary respectively. T_{ij} and U_{ij} are traction and displacement at an point in the domain for the point load considering each direction as independent. $r_{,i}$ and $r_{,j}$ are represent the radius derivatives with respect to (x) and (y) direction respectively. y_i is distance vector between integral point P and the rotation axis.

Fig.3 shows silicon nitride ceramic blade mesh, which was used for BEM analysis.

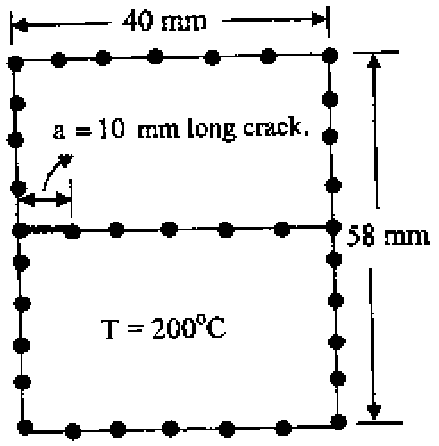

Fig.3 Boundary element mesh for blade and dimensions.

7- Result and Discussions

Fig.4 shows the risk of rupture (probability of failure) for ceramic disc under rotational loading only. The curves are obtained by drawing the risk of rupture obtained from Eq.10, Eq.12, and that obtained by Licht H. R. [3]. The curves show there is a good agreement between them. As indicated, the risk of rupture decreases with increasing tensile strength of material.

Fig.5 shows the expected values of rupture of rotating speed against radius. It can be seen the fracture speed decrease with increasing radius. As shown the expected values of $E(\omega^2)$ is remained constant at radius about 120 mm to outer radius.

Fig.6 and Fig.7 show the risk of rupture and expected values of rupture rotating speed under rotation and thermal loading. Also shown in same figures for comparison purposes the analogous values obtained by Duffy S. F [6]. The results show good agreement and the expected values of $E(\omega^2)$ is remained constant at radius 100 mm to outer radius.

Fig.8 and Fig.9 show the risk of rupture and expected values of rupture rotating speed under rotation and thermal loading for ceramic blade. These values are calculated for $\sigma_{max.}$ = 743.3MPa which is obtained from boundary element method. For comparison purposes, the analogous values obtained by Lich H. R. [3]. are plotted in same figures. The results show good agreement. As indicated the values of $E(\omega^2)$ is larger from that for ceramic disc.

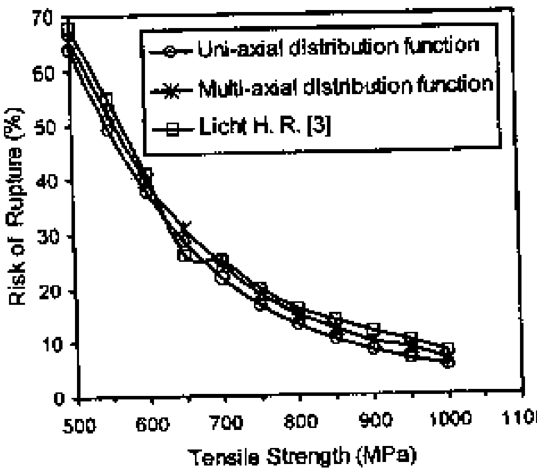

Fig.4 Risk of rupture for ceramic disc under rotation loading only.

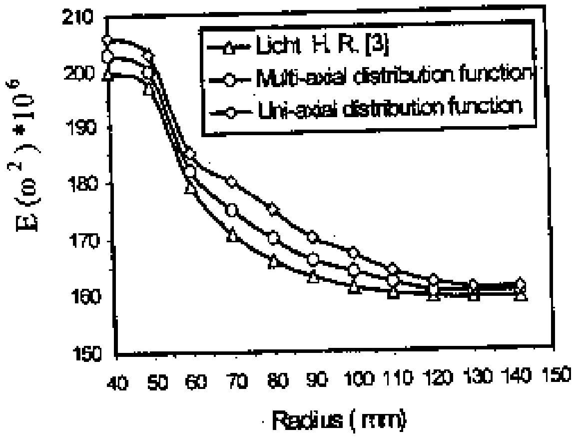

Fig.5 Expected values of ω^2 for ceramic disc under rotation loading only.

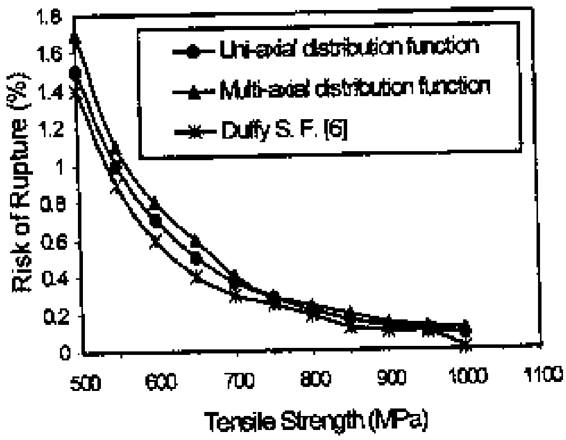

Fig.6 Risk of rupture for ceramic disc under rotation and thermal loading.

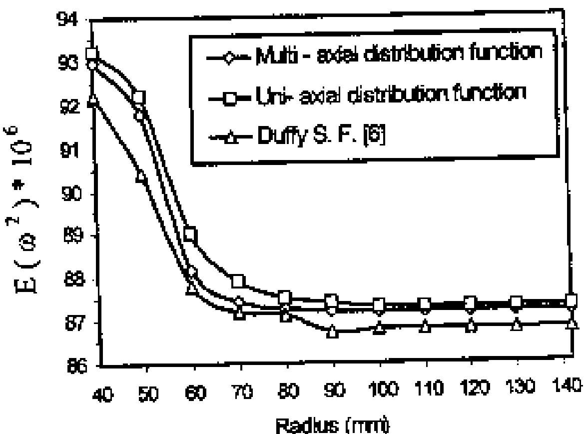

Fig.7 Expected values of ω^2 for ceramic disc under rotation and thermal loading.

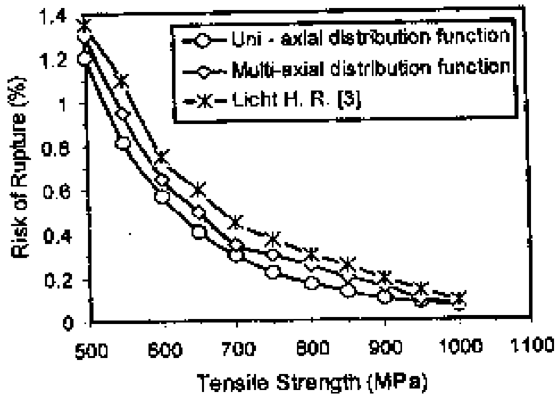

Fig.8 Risk of rupture for ceramic blade under rotation and thermal loading.

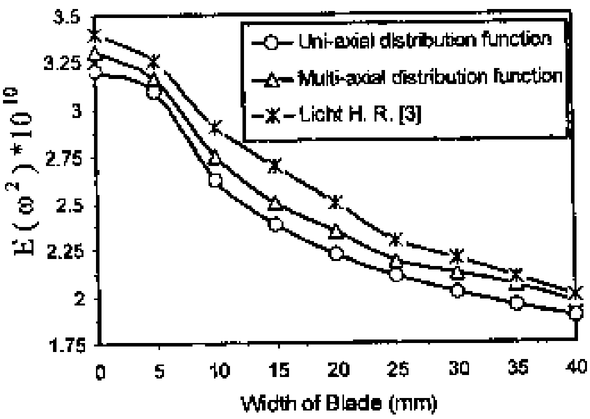

Fig.9 Expected values of ω^2 for ceramic blade under rotation and thermal loading.

8- Conclusions

From previous discussion the following conclusions are obtained:

1-Weibull multi-axial distribution function is developed and applied in ceramic turbine wheel taking the effect of existing crack and give more precise results compared to uni-axial distribution function results.

2- The risk of rupture and expected values of (ω^2) under rotation and thermal loading is less then that under rotation loading.

3- The expected rupture strength of ceramic blade is higher than of that of disc material, therefore the failure occurs in blade first than in disc material in service survival.

4- The approaches used in analysis can be extending to take the effect of residual stresses.

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